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Work performed under this program consisted of (a) consultative work with Wright-Patterson Air Force Base, (b) joint work with WPAFB to develop a concept for converting YBCO films deposited on a wide substrate strip into a conductor with a pattern of narrow spiral filamentary strips for ac operation, (c) calculation of the additional losses due to substrates on YBCO conductors and sheaths on BSCCO type conductors, and (d) new loss theory for polycrystalline high-T_c superconductors based on Maxwell's equations.

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FINAL TECHNICAL REPORT

For 01 Mar. 1995—28 Feb. 1998

WJC Research and Development
Contract # F49620-95-0016
P.I. W. J. Carr, Jr.
Program Manager: Dr. Harold Weinstock

(1). STATEMENT OF WORK:

WJC Research and Development will perform fundamental theoretical studies on AC losses in high-T_c superconductors, designed to advance the use of high-T_c superconductors in power devices such as rotating machinery and micro-SMES energy storage systems. Included in the studies will be studies of the critical current densities needed to compute the loss, and the effect of conductor geometry. The objective is to make significant progress in contributing to the understanding of high-T_c materials, bringing this understanding to the level which now exists for low-T_c materials. Results from this program which impact on conductor development will be regularly communicated to Oberly's group at Wright-Patterson Air Force Base. All work on the proposed program will be done by W. J. Carr, Jr. at 1460 Jefferson Heights, Pittsburgh, PA, 15235.

(2) Results of Research Effort

(a) On YBCO

Textured YBCO conductors as they are presently conceived for use at liquid nitrogen temperature (by either the IBAD or RABiTS method of preparation) are in the form of thin films of YBCO deposited on a substrate in the form of a wide strip. Thus their use would be limited to dc applications, since in ac magnetic fields a very large loss would result for perpendicular fields. The objective of work on YBCO in this program was to develop a concept which allows the possibility of ac use. The concept which was developed together with Charles Oberly at WPAFB was to convert the wide strip of YBCO film on the substrate into a spiral pattern of separated narrow strips as follows. Starting with a strip conductor produced by either the IBAD or RABiTS technique, having a width the order of 1 cm, groves are produced in order to give a pattern of separated narrow parallel strips of YBCO. The separate strips might typically be about 50 μm wide, separated by some fraction of this distance. The groves can be introduced by etching, mechanical scratching, laser ablation or (if it is found to not affect the texture) by introducing such a pattern into the original deposition through the use of a mask. (For a very thin YBCO film on the substrate one can consider stacking several substrates with their respective films on top of each other, but for a thick film one substrate strip may be sufficient for some problems). The substrate strip is then inserted between the two halves of a thin-walled split tube (made to have a relatively high resistivity) which when the two halves are clamped together gives mechanical support for the substrate. The resulting conductor is then slightly twisted to form a spiral filamentary-strip pattern of YBCO. Possibilities exist for internal cooling by passing liquid nitrogen through the tube.

(b) On BSCCO

Work on BSCCO superconductors under this program has been focused on developing a more accurate loss theory for high-T_c superconductors. Although much theoretical work has already been published, most of this work lacks the rigor which is found in similar loss calculations for ordinary low-T_c materials, and some is quite questionable. The approach which is used in the study of high-T_c superconductors usually depends on the objectives of the study. If the objective is to develop improved low-loss materials the loss is frequently viewed in terms of ad hoc mechanisms (such as vortex motion or the hysteretic behavior of multiple Josephson junctions) assumed to be responsible for the heating in particular cases. However, if the objective is simply to develop accurate loss expressions so that the correct amount of cooling can be provided, a general approach using Maxwell's equations is usually the better approach. The latter is the approach emphasized here. Maxwell's equations lead to more rigorous results because they give the loss by simply computing the work done on the superconductor by outside sources of power. Then thermodynamics can be used to compute the heating, independent of the individual internal mechanisms which lead to the heating.

Near the beginning of the twentieth century it was shown by H. A. Lorentz that Maxwell's equations (for normal materials) could be derived by averaging a more

fundamental set of Maxwell-Lorentz equations over atoms and molecules. The viewpoint I have always adopted in the past is that for a superconductor the same approach applies, except that additional and larger units of structure exist, and therefore the averaging must be made over a larger local volume element, big enough to enclose the larger structure. In the extreme case of filamentary structure this approach has been called the anisotropic continuum model, and it works well for both filamentary and non-filamentary low-T_c superconductors. One of the principle differences between a low- and a high-T_c superconductor is that in the latter, grain boundaries form an appreciable barrier to current flow. Thus, in a polycrystalline high-T_c case a new and important unit of structure exists in the crystal grains, and the average over the grains leads rigorously to the well-known concept of two critical current densities: the inter-grain critical current density and the intra-grain critical current density. The inter-grain critical current density corresponds to the critical current density of an ordinary low-T_c material and leads to a similar loss expression when the intra-grain circulating current can be neglected. However, in general the intra-grain current density is a second source of loss, and the two losses interact with each other. A loss theory has been developed based on this point of view, and reported on at both the 1997 and 1998 ICMC conferences. The results are given in the attached manuscripts in the Appendix.

(c) Loss in Sheaths and Substrates

At the request of the Wright-Patterson Air Force Laboratory the ac losses which can occur in the silver or silver-alloy sheath which exists on a BSCCO conductor, and the Ni alloy substrate on a typical YBCO conductor were computed. These losses were found to be small enough to be usually tolerated although not always negligible.

(3) List of Publications

Theory of Cyclic Loss in a High-T_c Superconductor, presented at the ICMC 1997.

Sheath and Substrate Losses in High-T_c Superconductors, presented at the ICMC 1997.

Toward a More Rigorous Understanding of AC Loss in a High-T_c Superconductor, presented at the ICMC 1998.

Filamentary YBCO Conductors For AC Applications—(with C. E. Oberly), to be presented at the Applied Superconductivity Conference 1998.

The manuscripts are collected in the Appendix.

(4) Professional Personnel

W. J. Carr, Jr.

(5) Interactions (Coupling Activities)

(a) Wright-Patterson Air Force Base

As required by the work statement, close contact was maintained with Charles Oberly at the Wright-Patterson Air Force Base, which included consultation and joint work.

(b) Ohio State University

Because of mutual interests a close consultative coupling was maintained with E. W. Collings and M. Sumption at Ohio State University. The area of mutual interest was the eddy current or coupling loss in BSCCO multifilament conductors.

(6) <u>Inventions</u>

No patent applications have been filed.

(7) <u>Information For Assessment of Program</u>

Roughly one-half the work on this program was directly related to a specific Air Force objective: the development of a a light-weight superconducting generator. The remaining work had the more general but still practical goal of predicting more accurately the loss in high-T_c superconductors, allowing an accurate estimation of the cooling requirement for any given application.

APPENDIX

To appear in proceedings for ICMC '97

ICMC
7/29/97

THEORY OF CYCLIC LOSS IN A HIGH-T_c SUPERCONDUCTOR

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ABSTRACT

High-temperature superconductors tend to require more carefully developed loss theory than ordinary low-T_c materials. For this reason expressions for the energy loss in a superconductor, based on Maxwell's equations, are re-derived in order to point out differences and similarities in the application of these results to high- and low-T_c materials.

INTRODUCTION

For a thermodynamic system which is repeatedly cycled, a cyclic state can be achieved which is the same at the end of a cycle as at the beginning. Thus, since the energy is unchanged it follows that $\Delta W = -\Delta Q$ where ΔW is the work done on the system and ΔQ is the heat added during the cycle ($-\Delta Q$ the heat removed). The latter is the "energy loss" per cycle.

The recent development of high-T_c superconductors has introduced a host of new problems in loss computation. The purpose of the present analysis is not to present completed solutions to these problems, but rather to outline ways in which such problems can be investigated, starting from an "exact" approach and some new points of view. All loss computations are given for the case where loss is produced by a cyclic applied magnetic field operating on an isolated body, and the frequencies assumed are those in the range of typical power systems. The exact ways of computing loss all involve the use of Maxwell's equations, as they pertain to superconductors, and it is important to understand the meaning of Maxwell's equations in a superconductor. In particular, it is important to understand the general nature of magnetization in superconductors, and the difference between effective and true magnetization. For continuity of argument a derivation of Maxwell's equations previously given by the Author is repeated here.

Fortunately, magnetostriction is very small in a superconductor, and the volume can be treated as constant. Therefore, work done on the system implies magnetic work. Consider the chain of events leading up to the loss: an external source of power performs work via a change in its current and magnet field, and if the magnetic field overlaps

the superconductor, part of this work goes into changing the instantaneous state of the superconductor. Then if a cyclic condition at a given temperature has been established, the net work done on the superconductor is transformed by internal mechanisms into heat to be carried away by the coolant. In analogy with the fact that loss can be measured in two ways (from the work or from the heat) the loss can be computed either from the work expended by the external source or from a computation based on the assumed mechanism behind the heat generation. The former is more rigorous, since the latter is tied to models for vortices, pinning forces and, in some analysis, to the behavior of Josephson junction arrays. Only the former approach is discussed here, but of course, both methods can be useful.

Since S.I. units are not convenient for the analysis used here, Gaussian units are used throughout.

EXACT LOSS COMPUTED FROM THE MAXWELL-LORENTZ EQUATIONS

Underlying Maxwell's equations is a set of Maxwell-Lorentz equations[1,2,3,4], which describe electromagnetism in terms of two "microscopic" fields: an electric field $\vec{e}$ and a magnetic field $\vec{b}$ (in the older literature the magnetic field is denoted by $\vec{h}$). The Maxwell-Lorentz equations are

$$\begin{aligned} \operatorname{curl} \vec{e} &= -\frac{1}{c}\frac{\partial}{\partial t}\vec{b} && (1) \\ \operatorname{div} \vec{b} &= 0 && (2) \\ \operatorname{curl} \vec{b} &= \frac{4\pi}{c}\vec{i} + \frac{1}{c}\frac{\partial}{\partial t}\vec{e} && (3) \\ \operatorname{div} \vec{e} &= 4\pi\rho_{\text{mic}} && (4) \end{aligned}$$

in terms of the microscopic current density $\vec{i}$ and the charge density ρ_{mic}, in otherwise obvious notation. To shorten the discussion, the case will be considered where no atomic orbital or spin currents exist in $\vec{i}$, which implies that the above equations describe changes over distances large compared with atomic distances, but nevertheless on a scale small enough to include any non-atomic current structure in the superconductor. High-T_c superconductors do have atomic moments, but they tend to be anti-ferromagnetic, and therefore not likely to be important for the loss. (Furthermore, any atomic magnetization can be easily introduced later into the Maxwell equations). Otherwise, for the present purpose the equations can be considered exact. Forming the scalar product of (1) with $\vec{b}$ and the scalar product of (3) with $\vec{e}$, subtracting the results, and using a well-known transformation one obtains

$$\vec{i}\cdot\vec{e} + \frac{1}{8\pi}\frac{\partial}{\partial t}\left(b^2 + e^2\right) = -\frac{c}{4\pi}\operatorname{div}\vec{e}\times\vec{b}. \tag{5}$$

Consider a system containing a superconductor and a normal magnetizing circuit. An integration over all space (a.s.) causes the right-hand side of (5) to vanish, and gives

$$\int_{\text{mag}} \vec{i}\cdot\vec{e}\,dV + \int_{\text{s.c.}} \vec{i}\cdot\vec{e}\,dV + \frac{1}{8\pi}\frac{\partial}{\partial t}\int_{\text{a.s.}} \left(b^2 + e^2\right) dV = 0 \tag{6}$$

where V is volume and the first two integrals are over the normal magnetizing circuit and over the superconductor. Eq. (6) is exact, involving only the Maxwell-Lorentz

equations. However, further progress requires the use of additional relations, i.e. constitutive equations, which in the case of a superconductor are often highly approximate. But for simple normal materials the constitutive equation in the form of Ohm's law will be treated as exact. For a normal material containing a source of emf such as a battery, with an emf density denoted by $\vec{e}^{(e)}$, Ohm's law becomes generalized[5] to $\vec{\imath} = \sigma\left(\vec{e} + \vec{e}^{(e)}\right)$, and $-\vec{\imath}\cdot\vec{e}$ can be written as $\vec{\imath}\cdot\vec{e}^{(e)} - i^2/\sigma$, where σ is the conductivity. The first term, integrated over the volume, is the energy taken from the battery, and the second subtracts the Joule heating. Thus, the first term on the left in (6), shifted to the other side of the equation, is just the net power P supplied by the magnetizing circuit to the superconductor and electromagnetic field, and (6) becomes

$$P = \int_{\mathrm{s.c.}} \vec{\imath}\cdot\vec{e}\,dV + \frac{1}{8\pi}\frac{\partial}{\partial t}\int_{\mathrm{a.s.}} \left(b^2 + e^2\right) dV. \tag{7}$$

Integrating over a cycle causes the field term to vanish, leading to

$$W = \oint dt \int_{\mathrm{s.c.}} \vec{\imath}\cdot\vec{e}\,dV \tag{8}$$

which is also the loss $-Q$. The expression (8) is correct for both low- and high-T_c superconductors. However, unfortunately it gives the loss in terms of a rapidly varying local loss density expression, which quite often is not easy to use.

DERIVATION OF MAXWELL EQUATIONS

A simplified derivation appropriate for superconductors is given here[3,4]. For any type of averaging, denoted by $\langle\,\rangle$, which commutes with space and time differentiation, i.e. $\langle \mathrm{curl}\,\vec{e}\rangle = \mathrm{curl}\,\langle\vec{e}\rangle$, etc., one can replace each variable A in (1) to (4) with $\langle A\rangle$, where the average is now assumed to be a volume average, and two of the Maxwell fields are immediately defined by $\vec{B} = \langle\vec{b}\rangle$ and $\vec{E} = \langle\vec{e}\rangle$. For definiteness the volume average will be defined by[2,6]

$$\langle F\rangle = \int d\vec{r}'\, f(\vec{r}')\, F(\vec{r} - \vec{r}', t) \tag{9}$$

where the function $f(\vec{r}')$ is approximately constant over a volume element ΔV, falls rapidly to zero outside ΔV, and integrates to unity. It is easily shown that this type of average does commute with differentiation.

For several reasons, the average of $\vec{\imath}$ is not simply the Maxwell current density $\vec{\jmath}$, with the principal reason being that the small scale description of current density $\vec{\imath}$ can be divided into a part which runs right through ΔV, and another part which approximately circulates within ΔV. The average of the first part defines $\vec{\jmath}$, and the second part defines a magnetization $\vec{M}$, which is described in detail in Ref. 3. It follows from a more careful analysis[3] that

$$\langle\vec{\imath}\rangle = \vec{\jmath} + c\,\mathrm{curl}\,\vec{M} + \frac{\partial}{\partial t}\vec{P} \tag{10}$$

where $\vec{P}$ is polarization[1], and the last term on the right is polarization current density. Since polarization is of much less interest than magnetization, it will simply be stated that

$$\langle\rho_{\mathrm{mic}}\rangle = \rho - \mathrm{div}\,\vec{P} \tag{11}$$

where ρ is the Maxwell charge density. Putting these results into the averaged equations (1) to (4) gives

$$\begin{aligned}
\operatorname{curl}\vec{E} &= -\frac{1}{c}\frac{\partial}{\partial t}\vec{B} && (12)\\
\operatorname{div}\vec{B} &= 0 && (13)\\
\operatorname{curl}\vec{B} &= \frac{4\pi}{c}\left(\vec{j}+c\operatorname{curl}\vec{M}+\frac{\partial}{\partial t}\vec{P}\right)+\frac{1}{c}\frac{\partial}{\partial t}\vec{E} && (14)\\
\operatorname{div}\vec{E} &= 4\pi\left(\rho-\operatorname{div}\vec{P}\right) && (15)
\end{aligned}$$

Defining a magnetic field $\vec{H}$ by $\vec{H} = \vec{B} - 4\pi\vec{M}$ and an electric displacement field $\vec{D}$ by $\vec{D} = \vec{E} + 4\pi\vec{P}$ allows the last two equations to be written in the compact Maxwell notation

$$\begin{aligned}
\operatorname{curl}\vec{H} &= \frac{4\pi}{c}\vec{j}+\frac{1}{c}\frac{\partial}{\partial t}\vec{D} && (16)\\
\operatorname{div}\vec{D} &= 4\pi\rho. && (17)
\end{aligned}$$

The purpose of reviewing this derivation is to point out the fact that Maxwell's equations are unique only in form. Each choice of the volume element ΔV can define a new set of equations having different values of $\vec{M}$, $\vec{j}$, and macroscopic fields[7].The magnetization $\vec{M}$ in Maxwell's equations is the true magnetization, which differs from what is commonly called "magnetization", i.e. the total magnetic moment divided by the total volume.

LOSS COMPUTED FROM MAXWELL'S EQUATIONS

Taking the scalar product of both sides of (16) with $\vec{E}$, and subtracting the scalar product of (12) with $\vec{H}$, and using a vector identity, leads to the well-known result

$$4\pi\vec{j}\cdot\vec{E}+\vec{H}\cdot\frac{\partial}{\partial t}\vec{B}+\vec{E}\cdot\frac{\partial}{\partial t}\vec{D} = -c\operatorname{div}\vec{E}\times\vec{H}. \qquad (18)$$

Consider again the system consisting of a normal magnetizing circuit and a superconductor. For an integral over all space the term on the right vanishes and one can write in analogy with (6)

$$\int_{\text{mag}}\vec{j}\cdot\vec{E}\;dV+\int_{\text{s.c.}}\vec{j}\cdot\vec{E}\;dV+\frac{1}{4\pi}\int_{\text{a.s.}}\left(\vec{H}\cdot\frac{\partial}{\partial t}\vec{B}+\vec{E}\cdot\frac{\partial}{\partial t}\vec{D}\right)dV = 0 \qquad (19)$$

An important point is that outside the superconductor no difference exists between $\vec{i}$, $\langle\vec{i}\rangle$ and $\vec{j}$, assuming a simple normal material with no magnetization or polarization in the magnetizing circuit. Likewise no difference exists between $\vec{e}$ and $\vec{E}$, and the integral of $-\vec{j}\cdot\vec{E}$ over the normal circuit is the same as the integral of $-\vec{i}\cdot\vec{e}$, or just the power P computed for the Maxwell-Lorentz equations. Then from (19)

$$P = \int_{\text{s.c.}}\vec{j}\cdot\vec{E}\,dV+\frac{1}{4\pi}\int_{\text{a.s.}}\left(\vec{H}\cdot\frac{\partial}{\partial t}\vec{B}+\vec{E}\cdot\frac{\partial}{\partial t}\vec{D}\right)dV. \qquad (20)$$

Integrating over a time cycle gives

$$W = \oint dt\int_{\text{s.c.}}\vec{j}\cdot\vec{E}\,dV+\oint dt\int_{\text{s.c.}}\left(\vec{H}\cdot\frac{\partial}{\partial t}\vec{M}+\vec{E}\cdot\frac{\partial}{\partial t}\vec{P}\right)dV \qquad (21)$$

where in a typical superconductor the polarization term can be neglected, which gives after setting $-Q$ equal to W

$$-Q = \oint dt \int_{\text{s.c.}} \vec{j} \cdot \vec{E}\, dV + \oint dt \int_{\text{s.c.}} \vec{H} \cdot \frac{\partial}{\partial t} \vec{M}\, dV\ . \tag{22}$$

While it might be thought that the loss computed from the large scale Maxwell equations is different from that of (8), the above derivation shows otherwise. As long as the superconductor is large enough for the averages to have meaning, the two expressions give precisely the same loss, and both expressions have their uses. The relative values of the two terms in (22), and the choice of ΔV, are the main features which distinguish high- from low-T_c loss theory.

THE CHOICE OF ΔV

As already noted, an arbitrary feature of the Maxwell equations is the volume element ΔV which defines the average. ΔV can be a sphere, or any shape and size which is small compared with the dimensions of the conductor. The number of useful choices for ΔV is determined by the number of structural features in the system. In a simple normal material only atomic structure (neglected here) is important, but such is not the case for a superconductor. In the mixed state of an ordinary superconductor, vortex structure exists in addition to atomic structure, and the simplest description is obtained for ΔV large enough to average out the vortices. But in a high-T_c superconductor, because of high anisotropy and weak links, the size, orientation, and shape of the crystallites are important structural features. Within each crystallite current can circulate to form a local magnetic moment, and additional local moments can possibly be established by current percolating around loops formed by crystallites of nearly the same orientation among randomly oriented crystals. By choice of ΔV one or more, or all, of these types of circulating current can be averaged out of the Maxwell current density $\vec{j}$. As long as the dimensions of ΔV are small compared with the conductor dimensions, the bigger ΔV, the smoother $\vec{j}$, with the tortuous path of $\vec{\imath}$ accounted for by the magnetization $\vec{M}$ it produces. The current density $\vec{j}$ is in its simplest form when all the local internal current structure has been averaged out.

Although the above discussion pertains to a monolithic superconductor, it can be extended to good approximation (anisotropic continuum model) to include filamentary conductors by simply making ΔV large enough to include filaments.

EFFECTIVE "MAGNETIZATION" AND TRUE MAGNETIZATION

The total magnetic moment of any isolated body is given quite generally by

$$\vec{m} = \frac{1}{2c} \int \vec{r} \times \vec{\imath}\, dV + \vec{m}_{\text{atomic}} \tag{23}$$

where $\vec{\imath}$ can be divided into various non-atomic parts, some being local and some non-local. In most methods of measurement only the total magnetic moment is measured, and $\vec{m}/V$ is called the "magnetization". However, this "magnetization" is not a true magnetization (which is roughly a local moment divided by a local volume) and it does not appear in Maxwell's equations. The above measured magnetization depends only on time, whereas magnetization in the sense of Maxwell and Lorentz is defined at each

point in space. Only currents which circulate on a scale the order of ΔV or less qualify as true magnetization currents. In a "soft" superconductor (no pinning)

$$\vec{i} = i_{\text{surface}} + \vec{i}_{\text{vortex terms}} \tag{24}$$

and it is convenient to take ΔV large enough to average out the variations due to vortex terms. Thus the vortex terms are the only true magnetization currents in a soft superconductor. The same remains true in an ordinary superconductor when pinning exists, since the additional current density which must be added to (24), i.e. a bulk (body) term $\vec{i}_{\text{bulk}}$ (average value $\vec{j}_{\text{bulk}}$) produced by pinning is not generally pictured as leading to appreciable localized circulating currents. To be significant (i.e. to produce a magnetic moment comparable with that of any transport-like current) magnetization current density must usually be quite large compared with the transport current density, because the former circulates within a small volume, and the latter over the whole conductor.

The Author has suggested[4] that the commonly measured "magnetization", which includes the magnetic moments of both local and non-local currents, should be called the effective magnetization. The effective magnetization is quite useful, and it can be used to compute the total loss, but not to discuss various details of the loss.

BEAN'S ASSUMPTION FOR ORDINARY HARD SUPERCONDUCTORS

Bean's contribution to the understanding of hard superconductors is usually thought of as the assumption of constant critical current density. However, his contribution, including some tacit assumptions, goes much deeper. In a strongly pinned hard superconductor Bean[8] assumed, tacitly, that the terms in (24) can be ignored compared with the bulk term $\vec{i}_{\text{bulk}}$. A full statement of Bean's assumption is $<\vec{i}>\approx\vec{j}$ (not necessarily true for high-T_c superconductors) and

$$\vec{j} \approx \vec{j}_{\text{bulk}} = j_c \frac{\vec{E}}{|\vec{E}|}. \tag{25}$$

In this approximation no true magnetization exists (the discussion does not include multifilament conductors) and the loss expression (22) reduces to just the $\vec{j}\cdot\vec{E}$ term, a result that usually applies quite accurately in ordinary low-T_c material.

THE CASE OF HIGH-T_C SUPERCONDUCTORS

Let it again be assumed, as Bean did for ordinary superconductors, that features such as surface and vortex currents can be neglected. Then the only structure to consider is the grain structure, and one can write

$$\vec{i} \approx \vec{i}_{\text{intergrain}} + \vec{i}_{\text{intragrain}} \tag{26}$$

where the intergrain part runs through the crystallites, and the intragrain part circulates within individual crystal grains. The obvious choice of ΔV is a value larger than the grain size (and for some problems considerably larger), which is assumed here. Then since one can always write

$$i = i_{\text{transport}} + i_{\text{magnetization}} \tag{27}$$

the intragrain current is all magnetization current while the intergrain part can make contributions to both terms in (27), with the magnetization part of the latter coming from local percolation currents on the scale of ΔV.

Two critical current densities can be defined: j_c^{inter} and j_c^{intra}. However, only the intergrain part can be described as in (25), since the direction of the intragrain current depends on the local electric field inside a grain. The only transport-like current comes from the intergrain part j_c^{inter}, which plays the role of j_c in low-T_c materials, and in regard to this current the high-T_c superconductors should behave in the same way in Maxwell theory as the low-T_c case, i.e. a boundary between regions of $\pm j_c^{\mathrm{inter}}$ (complicated here by the fact that, at least on one side, j_c^{inter} can decay in time[9]) forms with each half-cycle at the conductor boundary and moves in[4]. The loss produced by this boundary motion is computed from the $\vec{j}\cdot\vec{E}$ term in (22) with j given by j_c^{inter}. Thus when the $\vec{j}_c^{\mathrm{inter}}\cdot\vec{E}$ loss is dominant an approximate H^3 dependence should be observed for partial penetration, and an H dependence for full penetration, as indeed has been observed[10,11]. However, when magnetization terms such as that of the intragrain circulating current or that of percolation loops are important, the high-T_c case is quite different from the low-T_c case, and the full expression for the loss given by (22) must be used.

The magnetization terms in a high-T_c superconductor are illustrated in Fig. 1 , including the case of a multifilament conductor. Kwasnitza and Clerc[12] have estimated the loss due to intragrain circulating currents by applying low-T_c results to each grain, which may be adequate for elongated grains but the precise expression for this loss is given by computing the true magnetization for use in (22). However, when the loss due to j_c^{inter} is negligible the expression (8) can sometimes be used to good advantage to compute the intragrain loss.

ALTERNATE LOSS EXPRESSIONS

Another exact expression for the loss is given by the Poynting vector method, which can be easily shown[4] to be simply a transformation of (22). Although in some cases the Poynting vector approach is quite useful, in typical loss problems (22) is the simpler method to use. Other methods such as the plot of effective magnetization vs. applied field, and the flux-applied magnetic field method are discussed in Ref. 4. The latter is mainly useful for one-dimensional problems.

All the exact methods for computing loss can be obtained from a transformation of the theory presented here, and all require a solution to Maxwell's equations. The particular approach which is simplest is generally the one which allows the simplest approximations to be used.

SUMMARY

Exact loss expressions due to a cyclic magnetic field acting on an isolated conductor are computed from Maxwell's equations and the first law of thermodynamics. In a superconductor, more than one set of Maxwell equations can be constructed, depending on the size of the volume element over which the underlying Maxwell-Lorentz equations are averaged. The most convenient choice is one where ΔV is large enough to average out all internal structure in $\vec{j}$.

The Maxwell-Lorentz equations, themselves, can be used to compute a loss per

cycle, giving

$$-Q = \oint dt \int_{s.c.} \vec{i} \cdot \vec{e}\, dV \,. \tag{28}$$

However, this expression is not always convenient to use. The general expression given by the Maxwell equations (in the absence of polarization) for energy loss per cycle is

$$-Q = \oint dt \int_{s.c.} \vec{j} \cdot \vec{E}\ dV + \oint dt \int_{s.c.} \vec{H} \cdot \frac{\partial}{\partial t} \vec{M}\, dV \tag{29}$$

which is shown to be precisely equal to (28) (where it is assumed in both cases that no atomic magnetization exists). For conventional superconductors the magnetization term in (29) can be neglected, but for high-T_c materials it can become important, which is the main distinction, loss-wise, between high-T_c and low-T_c superconductors. The distinction comes about because grain structure is much more important in a high-T_c superconductor, and in a non-textured material with large grains relatively large currents can circulate within the grains. Thus with the assumption of a large volume element which includes grains (and perhaps local percolation loops) for taking averages, these currents correspond to magnetization currents. The Maxwell current density j in this description is j_c^{inter}, usually a relatively weak value. However, when the $\vec{j}_c^{\text{inter}} \cdot \vec{E}$ part of the loss is dominant the loss is computed just as in the low-T_c case, i.e. a boundary forms between regions of $\pm j_c^{\text{inter}}$ and moves inward each half-cycle[4]. An added feature is that j_c^{inter} on one side of the boundary can decay in time while the boundary is moving (and also for full penetration).

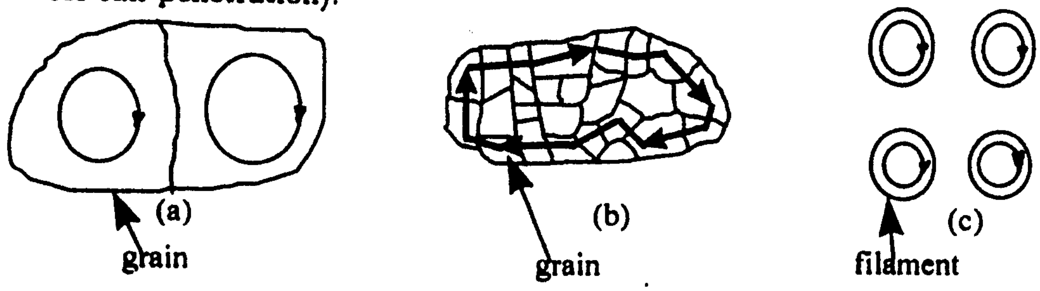

Figure 1. Magnetization current in high-Tc material: (a) intragrain, (b) current percolating among grains with nearly the same orientation (remains to be confirmed), (c) filaments in multifilament case.

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SHEATH AND SUBSTRATE LOSSES IN HIGH-T_C SUPERCONDUCTORS

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ABSTRACT

Metallic substrates and sheaths in practical high-T_c superconductors will produce extra losses in ac operation. These losses are investigated in general, and applied to the case of a Ag or Ag-Mg sheath, and a Ni-alloy substrate, for the range of frequencies used in power systems (order of 100 Hz). For generality a ferromagnetic substrate is considered, but in a thin tape the ferromagnetism is found to have little effect on the principal loss, which is an eddy current loss for a magnetic field applied perpendicular to the face of the tape.

INTRODUCTION

A common feature of several proposals for producing a superconducting tape of textured YBCO is the presence of a substrate which is sometimes metallic[1,2]. Other types of high-T_c conductors are contained within a Ag sheath[3]. In alternating magnetic fields the substrates and sheaths cause additional losses, apart from the loss in the superconductor itself, and a question of some interest concerns the limits which these additional losses set on the use of high-T_c conductors for ac operation. A computation of such losses is difficult only in the case of a tape with applied magnetic field normal to the face of the tape. However, for convenience, loss expressions for the simpler cases are also given and applied to the case of a Ag or Ag-Mg outer sheath and a Ni-alloy substrate. For generality the substrate is assumed to be ferromagnetic. Although the presence of the superconductor will have some effect, due mainly to shielding, for simplicity the calculations are made for an isolated substrate or outer sheath, which tends to give an upper limit on the sheath and substrate losses in a transverse applied field.

MAGNETIC FIELD AND FLUX DENSITY IN A THIN FERROMAGNETIC STRIP

The magnetic field $\vec{H}$ in a ferromagnetic material can be written as the sum of an externally applied field $\vec{H}_a$ (assumed to be uniform in space), a field produced by internal currents which will be labeled $\vec{H}_j$, and the demagnetizing field $\vec{H}_{demag}$ due to divergence in the magnetization. Thus the induction or flux density $\vec{B}$ is

$$\vec{B} = \mu_0(\vec{H}_a + \vec{H}_j + \vec{H}_{demag} + \vec{M}) \tag{1}$$

where $\vec{M}$ is the magnetization.

In-Plane Field

For an in-plane applied field in a very thin strip the demagnetizing field is small. Furthermore, the current induced by an ac in-plane applied field circulates within the thickness of the cross-section, and the field it produces is also quite small (assuming the thickness is less than the skin depth for the material). For simplicity the loss is considered for the case where the strip is sufficiently thin so that both H_{demag} and H_j can be neglected, and therefore for an in-plane field $\vec{B} \approx \mu_0(\vec{H}_a + \vec{M})$. The time rate of change $\dot{M}$ of the magnetization is $\dot{M} = \chi_d \dot{H} \approx \chi_d \dot{H}_a$ where χ_d is the differential susceptibility, and the time rate of change of the flux density for the in-plane field is approximately

$$\dot{B} = \mu_0(1 + \chi_d)\dot{H}_a . \tag{2}$$

If the z axis is along the length of the strip, the y axis along the thickness, and x along the width (Fig. 1) then $\vec{H}_a$ in this case is in the x or z direction, with the former (transverse case) of greatest interest, and the case considered here. (Actually, for a wide strip both cases give the same loss.) Consequently, $\dot{B}$ in (2) is in the x direction, while the electric field $\vec{E}$ it produces is in the z direction.

Perpendicular Field

In contrast, for an applied field perpendicular to the face of the strip all three contributions to $\vec{H}$ can be important. The induced current tends to flow along the z axis, but it changes over the width of the strip rather than over the thickness. A large demagnetizing field exists due to divergence of magnetization at the surfaces. However, little divergence of magnetization exists inside the strip because the magnetization is directed along the thickness, and not much change can occur over the small thickness, assuming no pronounced skin effect in this direction. For similar reasons all three magnetic field vectors are approximately constant through the thickness, and largely directed along the y axis. But since the demagnetizing field is approximately - M for the thin strip it nearly cancels M, and the flux density reduces to $B \approx \mu_0(H_a + H_j)$, which is the same as for a non-ferromagnetic strip. Although the above approximation does not hold near the edges of the strip, inasmuch as the strip does not have an exact demagnetizing factor, any attempt to include this fact would greatly complicate the problem. Thus it is assumed that for an applied field perpendicular to the face of the strip (y direction)

$$\dot{B} \approx \mu_0(\dot{H}_a + \dot{H}_j) \tag{3}$$

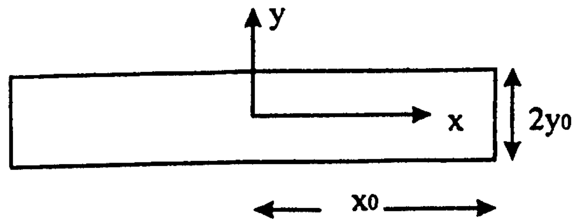

Figure 1. Cross-section of strip.

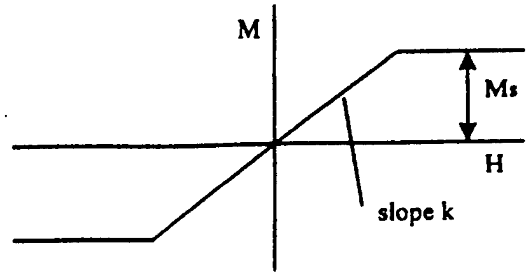

Figure 2. Assumed magnetization curve.

which is also in the y direction. In reality, the "applied" field should include the effect of the superconductor, but as mentioned in the Introduction the calculation will be made as if the superconductor were not present, which is an approximation that assumes full penetration of the superconductor over most of the time cycle.

In general for a transverse applied field it follows from symmetry that $\vec{H}_j$ and $\vec{B}$ have no z component, and since for a long strip ∂/∂z can be set equal to zero, the Maxwell equation curl $\vec{H}_j = \vec{j}$ becomes

$$\hat{z}(\partial H_{jy} / \partial x - \partial H_{jx} / \partial y) = \vec{j} \tag{4}$$

showing that in both transverse field cases $\vec{j}$ and $\vec{E} = \vec{j} / \sigma$ (σ the conductivity) are in the z direction. The Maxwell equation for curl $\vec{E}$ becomes

$$\hat{x}\partial E_z / \partial y - \hat{y}\partial E_z / \partial x = -\dot{\vec{B}}. \tag{5}$$

EDDY CURRENT LOSS IN THIN FERROMAGNETIC STRIP FOR TRANSVERSE IN-PLANE FIELD

Since in this case B is in the x direction and E is an odd function of y, and from (2) B depends only on time, it follows from (5) that

$$E = -\dot{B}y. \tag{6}$$

The current density j is equal to σE and the integral of $\vec{E} \cdot \vec{j}$ over the over the volume gives for the power loss P per unit volume V of the strip

$$\frac{P}{V} = \frac{\sigma}{3}\dot{B}^2 y_0^2 \tag{7}$$

with y_0 the half-thickness. To simply the calculation the magnetization curve shown in Fig. 2 will be assumed, where χ_d in (2) is either k or 0. For weak fields it is equal to k, while for strong fields and a sinusoidal time cycle

$$H_a = H_{a0} \sin \omega t, \tag{8}$$

over most of the cycle $\chi_d = 0$.

Weak Magnetic Field Acting on Ni-Alloy Substrate

For $H_{a0} < M_s / k$, (M_s the saturation magnetization, taken here to be about that for Ni in a dilute alloy) the differential susceptibility is the same as the susceptibility χ, and $1 + \chi_d$ in (2) is simply the relative permeability μ_r. Thus from (7) the time average loss divided by the volume becomes

$$\frac{<P>}{V} = \frac{\sigma}{6}\mu_0^2\mu_r^2 H_{a0}^2\omega^2 y_0^2. \tag{9}$$

The factor $\mu_0\mu_r H_{a0}$ in the weak field case is no more than the order of the saturation induction, which for Ni is about 0.6 T. Assume a value for σ of 10^7 mho/m and a frequency of 100 Hz. Then the right-hand side of (9) is about $2.4 \times 10^{11} y_0^2$ or less. For a thickness $2y_0 = 2.5 \times 10^{-5}$ m (about one mil) a very small value in the neighborhood of 40 W/m^3 is obtained, and one can conclude that the loss produced by a weak in-plane field is not likely to be a problem

Strong Magnetic Field Acting on Ni-Alloy Substrate

For $H_{a0} >> M_s/k$ one can replace the factor μ_r in (9) with unity, and

$$\frac{<P>}{V} \approx \frac{\sigma}{6}\mu_0^2 H_{a0}^2\omega^2 y_0^2. \tag{10}$$

Consider in this case $\mu_0 H_{a0}$ equal to 2 T, with the other parameters unchanged. The loss in this case is about 400 W/m^3, which is still quite small for most applications, and, clearly, only a perpendicular field is likely to cause excessive losses in a Ni-alloy substrate which is held to a thickness no more than several mils.

EDDY CURRENT LOSS FOR FIELD PERPENDICULAR TO PLANE OF STRIP

In this case it makes no difference whether the strip is ferromagnetic or not, as discussed previously. Although a very thin strip is essentially one-dimensional in cross-section, it must be treated as a limiting case of a two-dimensional problem, because the second derivative with respect to y will be found to be important even for y → 0. From curl $\vec{E} = -\dot{\vec{B}}$ the curl of both sides of the equation together with $\vec{E} = \vec{j}/\sigma$ gives $\nabla^2\vec{j} = \sigma curl\dot{\vec{B}}$. Then from $\vec{B} \approx \mu_0(\vec{H}_a + \vec{H}_j)$ it follows that $curl\vec{B} = \mu_0 curl\vec{H}_j = \mu_0\vec{j}$ and

$$\nabla^2\vec{j} = \mu_0\sigma\dot{\vec{j}} \tag{11}$$

which can be written alternatively as

$$\nabla^2 j = \frac{2}{\delta^2}\frac{1}{\omega}\dot{j} \tag{12}$$

where δ is the skin depth for a non-ferromagnetic material, given by

$$\delta^2 = \frac{1}{\mu_0}\frac{2}{\sigma\omega}. \tag{13}$$

B can be obtained in terms of j by using (5), i.e.

$$B_x = -\frac{1}{\sigma}\frac{\partial}{\partial y}\int j dt \tag{14}$$

$$B_y = \frac{1}{\sigma}\frac{\partial}{\partial x}\int j dt \tag{15}$$

and from the general Biot-Savart expression

$$\vec{H}_j = curl \frac{1}{4\pi}\iiint \frac{\vec{j}}{r''} dx'dy'dz'. \tag{16}$$

where $r'' = |\vec{r} - \vec{r}\,'|$.

The common approach to solving Maxwell's equations is to solve for $\vec{j}$ or $\vec{B}$ as a sum of terms with arbitrary coefficients, and the coefficients are determined from simple boundary conditions. Unfortunately, in the perpendicular field case no simple boundary conditions are evident. However, the equation

$$\frac{\vec{B}}{\mu_0} = \vec{H}_a + curl \frac{1}{4\pi}\iiint \frac{\vec{j}}{r''} dx'dy'dz' \tag{17}$$

evaluated inside the strip serves the purpose of boundary conditions, because the equation holds both inside and outside the strip. Thus in principle, the Maxwell equations are solved by obtaining a complete set of solutions for j from (12), and evaluating the coefficients from (17), using (14) and (15) for $\vec{B}$. Consequently, the exact "boundary" conditions are

$$-\frac{1}{\mu_0\sigma}\frac{\partial}{\partial y}\int j dt = \frac{1}{4\pi}\frac{\partial}{\partial y}\iiint \frac{j}{r''} dx'dy'dz' \tag{18}$$

and

$$\frac{1}{\mu_0\sigma}\frac{\partial}{\partial x}\int j dt = H_a - \frac{1}{4\pi}\frac{\partial}{\partial x}\iiint \frac{j}{r''} dx'dy'dz' \tag{19}$$

However, the integrals giving $\vec{H}_j$ are difficult to perform and an exact solution is very complicated. In order to simply the problem let the thickness be assumed to approach zero, and let j be approximated by a single term which satisfies (12). Let j be the real part of the complex expression

$$j = Ce^{i\omega t} x \cosh(1+i)\frac{y}{\delta} \tag{20}$$

where C is an arbitrary constant. The expression has the correct symmetry, being odd in x and even in y. For $y_0 << \delta$ this solution reduces to just a function of x and t, i.e.

$$j \approx Ce^{i\omega t}x \tag{21}$$

but only the full expression (20) satisfies (12), and the result could not have been obtained starting from a one-dimensional approximation in space.[4]

If the thickness of the strip is now allowed to approach zero both components of $\vec{H}_j$ approach zero, because the integral which gives $\vec{H}_j$ is proportional to the thickness. Furthermore, on the left-hand side of (18) $\frac{\partial j}{\partial y} \to 0$ (more rigorously, it can be shown that the two sides of (18) approach zero in the same way) and one is left with the equation

$$\frac{1}{\sigma}\frac{\partial}{\partial x}\int j dt = \mu_0 H_a \tag{22}$$

from (19). Writing $H_a = H_{a0}e^{i\omega t}$ and using (21) and (22) leads to the solution

$$C = \mu_0 i\omega\sigma H_{a0} \tag{23}$$

for a very thin strip. Thus for this case (21) gives

$$j = \mu_0 i\omega\sigma H_a x = \mu_0\sigma\dot{H}_a x\,. \tag{24}$$

The approximation holds up to the point where δ becomes comparable with y_0, i.e. where skin effect begins to occur through the thickness. The power loss due to (24) is

$$\frac{P}{V} = \frac{\sigma}{3}\mu_0^2\dot{H}_a^2 x_0^2\,, \tag{25}$$

and

$$\frac{<P>}{V} = \frac{\sigma}{6}\mu_0^2 H_{a0}^2\omega^2 x_0^2 \tag{26}$$

where x_0 is the half width. For a somewhat more favorable set of parameters than considered previously, assume a frequency of 50 Hz, $\sigma = 5\text{x}10^6$ mho/m and $\mu_0 H_{a0}$ equal to 1 T. Then for a strip width $2x_0 = 0.5$ cm the time average loss is 0.5 W/cm^3.

HYSTERESIS LOSS IN A Ni SUBSTRATE

Only the saturation magnetization case will be considered. If the hysteresis loop is approximated by a rectangle the loss density per cycle is $\mu_0 4H_c M_s$ (H_c the coercive force) and

$$\frac{<P>}{V} = 4\mu_0 f H_c M_s \tag{27}$$

with f the frequency. For f the order of 10^2 this loss for Ni is the order of 10^{-1} to 10^{-2} W/cm^3.The hysteresis loss is generally the largest in-plane field loss for a ferromagnetic substrate, but it can still be kept within allowable limits for most problems.

LOSS IN Ag AND Ag-ALLOY SHEATH

Flattened Sheath

For a conductor which has been flattened into a tape the results for a strip can be used. Thus for the perpendicular field case the loss value calculated for the Ni-alloy substrate, multiplied by the ratio of conductivities, applies. Since in the range of 77 K to 4.2 K the conductivity of Ag[5] is the order of 10^2 to 10^4 times larger than that assumed for the Ni alloy, a pure silver sheath is not well-suited for ac operation. But a Ag-Mg alloy[6] with σ in the neighborhood of 5×10^7 mho/m seems to be acceptable.

Circular Sheath

For a circular sheath the transverse field loss does not differ greatly from that for the perpendicular field loss for a strip if the diameter is the order of the strip width. A computation of the loss for a circular sheath gives

$$\frac{<P>}{V} = \frac{\sigma}{4} \mu_0^2 H_{a0}^2 \omega^2 R_0^2 \tag{28}$$

where R_0 is the radius. As in the case of a perpendicular field acting on a strip, the thickness of the sheath does not enter the expression for the loss per unit volume (as long as it is small compared with the radius).

SUMMARY AND DISCUSSION

The possibility of developing radical new geometries, or of sufficiently improving present geometries, to allow the use of high-T_c superconductors in the ac magnetic fields of power devices is a possibility which is receiving increased consideration[7]. One of the first questions one can ask involves the limitations imposed by sheaths and substrates. It is shown that for many applications the existence of a thin Ni-alloy substrate does not impose a serious limitation. The largest in-plane field loss for a ferromagnetic Ni-alloy substrate is generally the hysteresis loss. The largest eddy current loss, by far, is the loss for a field perpendicular to the plane of the substrate, but both these losses appear to be manageable.

On the contrary, a Ag sheath is not well suited for ac operation, but the use of a Ag-Mg alloy tends to lead to acceptable sheath losses.

For a ferromagnetic strip in a weak in-plane applied magnetic field the power loss per unit volume (SI units) is

$$\frac{<P>}{V} = \frac{\sigma}{6} \mu_0^2 \mu_r^2 H_{a0}^2 \omega^2 y_0^2 . \tag{9}$$

In a strong field for a ferromagnetic material, or in general for a non-ferromagnetic, the expression reduces to

$$\frac{<P>}{V} \approx \frac{\sigma}{6} \mu_0^2 H_{a0}^2 \omega^2 y_0^2 \tag{10}$$

where σ is the conductivity, ω is the angular frequency $2\pi f$, H_{a0} is the peak applied field, μ_r the relative (true) permeability, and y_0 the half-thickness. For the case of a field perpendicular to the face of a strip the loss is

$$\frac{<P>}{V} = \frac{\sigma}{6}\mu_0^2 H_{a0}^2 \omega^2 x_0^2 \tag{26}$$

with x_0 the half-width. The above equations for strips can be used for both thin substrates and thin flattened sheaths. The case of a circular sheath is given by (28). The saturation hysteresis loss for a ferromagnetic substrate is given by equation (27).

Equation (9) is the same as the standard loss expression for transformer laminations[8], (which differs only in the fact that the magnetic field is longitudinal rather than transverse) and (10) is the same expression for non-ferromagnetic laminations. Equation (26) is the same as (10) with the half-thickness y_0 replaced by the half-width x_0, but this similarity is misleading. The approximations used to obtain (26) are much different. In particular (26) applies without regard to whether the substrate is ferromagnetic or non-ferromagnetic. Furthermore, (10) comes from a one-dimensional problem in y and it applies for y small compared with the skin depth. The result (10) breaks down at the point where the half-thickness is equal to the skin depth. Although (26) involves only the one dimension x, its derivation requires the solution of a two-dimensional problem. Unlike the case for y_0 in (10), x_0 in (26) can be much larger than the skin depth. However, (26) like (10) breaks down when y_0 becomes comparable with the skin depth. The derivation of (26) given here is based only on Maxwell's equations, but it agrees reasonably well with previous work by Del Vecchio and Sefko[9] based on a variational calculation[10]. The present result gives values about 19% larger.

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Toward a more rigorous understanding of AC loss in a high-T_c superconductor

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Abstract

Although many useful expressions have been derived for the AC loss in high-T_c superconductors, most lack the rigor which exists in low-T_c expressions. One way to obtain more rigorous results is simply to apply Maxwell theory more carefully, since, in principle, Maxwell theory provides a systematic, exact approach for the AC loss in any material. High-T_c superconductors require special treatment only because they are much more complicated than low-T_c superconductors. A description of Maxwell loss theory as it applies to high-T_c superconductors was recently given by the author, and this approach is used here for the purpose of improving the understanding of hysteresis loss in these materials.

granular superconductivity, critical current density, magnetization

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Introduction

The approach which is used in the study of AC loss in high-T_c superconductors generally depends on the objectives of the study. For example, materials scientists with the goal of improved materials tend to view loss in terms of various internal heat producing mechanisms, involving vortices or multiple Josephson junction behavior. However, applied scientists who need to know the loss only to provide for cooling are more likely to take the rigorous viewpoint of Maxwell's equations. The present analysis is concerned with the latter, based on a derivation of Maxwell's equations in high-T_c superconductors given previously[1]. Only Maxwell's equations can lead to rigorous loss expressions, for they involve only the work done on a superconductor by external sources of power, and for cyclic work at a fixed temperature the heating follows from thermodynamics, quite independent of the particular internal mechanisms which produce it. Thus, for a macroscopic superconductor under cyclic electromagnetic perturbation Maxwell's equations lead to loss expressions which, in principle, are exact. Of course, it does not follow that an exact loss can be obtained from these expressions, for they ultimately require solutions of Maxwell's equations, which for a superconductor tend to be highly approximate. Nevertheless, in regard to understanding the loss some benefit accrues from starting with an exact expression. In the present analysis the principal differences between approximations for low- and high-T_c superconductors are discussed.

Exact loss expressions

For a superconductor carrying a net transport current and acted upon by an applied magnetic field the loss in the superconductor comes, in general, from two external sources of power: the power supply connected to the current leads, and the external source of power which produces the applied field. For simplicity only a uniform polycrystalline isolated body (no net transport current) will be considered here, and the entire power comes from the external magnetic field. The considerable number of equivalent ways that loss expressions coming from Maxwell's equations can be stated have been discussed previously[2]. The one treated in detail in Ref. [1] is used here. A microscopic expression which applies under all circumstances, obtained from the Maxwell-Lorentz rather than the Maxwell equations, is the rather obvious expression

$$-Q = \oint dt \int \mathbf{i} \cdot \mathbf{e} dV \tag{1}$$

where $-Q$ is the heat loss per cycle, $\mathbf{i}$ and $\mathbf{e}$ the microscopic current density and electric field, and the integrals are over time and volume V of the superconductor. However, the current density and electric field vary rapidly over volume in this expression, with loss occurring both in the grain boundaries and inside the grains. To obtain a macroscopic expression for the loss, the current density is first broken up into two parts: a transport-like part and a magnetization part, i.e.

$$\mathbf{i} = \mathbf{i}_{tr} + \mathbf{i}_{mag}. \tag{2}$$

One then defines a volume element ΔV (small compared with V) over which averages are taken. The transport-like current density runs right through ΔV (contributing to the total magnetic moment of the body but not to the local magnetization), while the magnetization part is the current density of a series of localized circulating currents, roughly the size of ΔV or smaller. In an average over (2) the magnetization current tends to average out unless these currents differ at different macroscopic points in the superconductor. In general the average of (2) is[1,2]

$$< \mathbf{i} >= \mathbf{j} + c\,curl\mathbf{M} \tag{3}$$

where $\mathbf{j}$ is the Maxwell current density (the average of $\mathbf{i}_{tr}$), and $\mathbf{M}$ is the true magnetization (Gaussian units are used). The size and shape of ΔV are taken such that $\mathbf{j}$ is a smooth macroscopic function with local variations mostly averaged out. In this regard the size of ΔV is very important, since its choice will be found to define one of the principal differences between the loss theory for low- and high-T_c superconductors.

The Maxwell equations come from averages over the Maxwell-Lorentz equations, which together with (3), the well-known averages $\mathbf{E}$ and $\mathbf{B}$ and the definition of $\mathbf{H}$ as $\mathbf{H} = \mathbf{B} - 4\pi\mathbf{M}$ lead to an AC loss per cycle given by[1,2]

$$-Q = \oint dt \int \mathbf{j} \cdot \mathbf{E} dV + \oint dt \int \mathbf{H} \cdot \frac{\partial}{\partial t} \mathbf{M} dV \tag{4}$$

where the only approximations are that the electric field energy is neglected compared with that of the magnetic field, any terms in polarization are neglected compared with those in magnetization, and any atomic magnetization is neglected. (Atomic moments do exist in most high-T_c superconductors, but they tend to be anti-ferromagnetically arranged). Equations (4) and (1) give the same loss, but approximations are easier to make using (4). It must be emphasized that the magnetization in (4) is not what is commonly called 'magnetization' (total magnetic moment divided by total volume, which will be called effective magnetization here) but the true magnetization of Maxwell and Lorentz (approximately the magnetic moment in ΔV divided by ΔV). In a low-T_c superconductor the true magnetization is usually neglected (the principal localized circulating currents are vortex currents which for the present purpose can be neglected), and $\mathbf{B} \approx \mathbf{H}$.

Interpretation of the loss and discussion of loss density

The loss expression (4) can be transformed to give a variety of new but equivalent expressions[2]. For example, one of these corresponds to the commonly used method of evaluating the loss from a plot of total magnetic moment (or on a per unit volume basis the 'effective magnetization') versus the applied field $\mathbf{H}_a$. However, such a plot gives no clue as to where the loss occurs. The important point to note is that different expressions involve different electromagnetic quantities and therefore provide different possible interpretations. Equation (4) is an expression involving the true magnetic field and the true magnetization, along with the Maxwell current density and electric field. Although

(4) gives the same total loss as the above mentioned plot, it also implies a macroscopic loss density. A microscopic loss density is implied in the expression (1).

Unfortunately, there is no rigorous way to derive the loss density in a superconducting body from Maxwell's equations, because one can always add a function which integrates to zero over the volume. However, in a normal material the power density is given by $\mathbf{j} \cdot \mathbf{E}$, and since Eq. (4) implies a density which is just an extension of the normal material expression, it is tempting to assume that (4) gives the correct macroscopic power density. A confirmation of this assumption would require measurement of the temperature gradients within the superconductor. It is possible that a knowledge of the correct loss density would in some cases be of interest in stability theory.

For a high-T_c superconductor the viewpoint of Eq. (4) , as discussed below, is that a loss occurs (computed from $\mathbf{j} \cdot \mathbf{E}$) due to macroscopic inter-grain current, along with an additional loss due to the true magnetic field acting on the true magnetization produced by circulating current in each grain. The latter will depend on the position of the grain in the conductor.

Differences between loss theory for high- and low-T_c materials

In ordinary low-T_c superconductors, as mentioned above, the true magnetization can usually be neglected, and from (4)

$$-Q \approx \oint dt \int \mathbf{j} \cdot \mathbf{E} dV . \tag{5}$$

This approximation does not generally extend to high-T_c superconductors because in the latter, grain boundaries (unless the grains are closely aligned) form a barrier to current flow. Large current densities can be induced to circulate within the crystal grains (if sufficient pinning exists), but only a small density of current can flow across the grain boundaries. Thus, in (2) the magnetization current density tends to be large in magnitude compared with the transport part, and although its average value tends to be small because of sign changes, the magnetization produced by the intra-grain circulating current can be appreciable, particularly if the grains are large. Therefore, in order to include this intra-grain magnetization, and also to give a smooth value of $\mathbf{j}$, the volume element for averaging must be taken to be at least the order of the grain size. (In Ref. [1] it was suggested that currents circulating around a path following well-aligned grains might also produce an appreciable true magnetization, but this possibility will not be considered here).

It follows that significant differences between high- and low-T_c loss theory exist as follows:

(a) High-T_c material with large grains not completely aligned can have an appreciable true magnetization, produced by intra-grain circulating current, and the volume element ΔV over which averages are taken is determined by the grain size.

(b) The loss in low-T_c material can be approximated by (5) whereas for high-T_c superconductors the full expression (4) must be used.

Other differences, which are already well-known, are

(c) Two critical current densities can be defined in high-T_c superconductors.

(d) Large anisotropy exists in these critical current densities.

(e) Critical current density is more subject to decay.

(f) Larger dc loss can occur in high-T_c material above a critical electric field.

But similarities occur in that Bean models and modified Bean models can be used in both cases. In the high-T_c case the two critical current densities are associated with the two types of current densities in (2). In a Bean-type model let $j_c^{\mathrm{int}\,er}$ be a critical value for the average transport-type current density j at any point, i.e. j can take only the values $j_c^{\mathrm{int}\,er}$ or zero. Then $j_c^{\mathrm{int}\,er}$ corresponds to what is called j_c in low-T_c superconductors, and when anisotropy, magnetization current and time decay are all unimportant, one can simply take over the familiar hysteresis loss results for low-T_c material, replacing j_c with $j_c^{\mathrm{int}\,er}$. The same is true of modified Bean models where $j_c^{\mathrm{int}\,er}$ becomes a function of H.

The magnetization loss in the high-T_c superconductor tends to become more important for the case of large grains which are poorly aligned. The poor alignment makes $j_c^{\mathrm{int}\,er}$ very small and less important (relatively high temperatures and fields make $j_c^{\mathrm{int}\,er}$ even smaller), and the large grains tend to make the intra-grain magnetization large (assuming good pinning within the grain). The second critical current density comes into play in the computation of the magnetization. In Eq. (2) one can take the magnitude of

$\mathbf{i}_{mag}$ to have two values: j_c^{intra} and zero. Then for a given grain with volume ΔV_g the magnetization of the grain is (approximately)

$$\mathbf{M} = \frac{1}{\Delta V_g 2c} \int \mathbf{r} \times \mathbf{i}_{mag} dV \tag{6}$$

where the integral is over the volume of the grain, and the critical current density j_c^{intra} can be used in evaluating the integral. Clearly, because of anisotropy the magnetization will depend on the orientation of the magnetic field relative to the c crystallographic axis.

In regard to the time decay in this picture, one should not expect j_c^{inter} and j_c^{intra} to decay at the same rate[3]. The former should be determined by conditions in the grain boundaries, while for large grains the latter should behave much like a single crystal.

Results for a high T_c slab with slab-like grains for an in-plane transverse field

Consider the familiar problem of a slab in a transverse applied magnetic field H_a, in the plane of the slab (see Fig. 1). As in the case of low-T_c superconductors the critical current density j_c^{inter} penetrates in from both surfaces on each half cycle, having opposite signs on the two sides. The problem will be greatly simplified by neglecting any time decay of the critical current density and using a strict Bean model of constant critical current density. (Since, in fact, the critical current density depends strongly on magnetic field, judgment must be used to choose a value appropriate for the given maximum field).

The grains will be assumed to be flat, i.e. also slab-like in shape, with the c axis normal to the plane of the grain, and slightly tilted with respect to the plane of the large slab. For simplicity it will be assumed that the tilt angles of neighboring grains differ from each other enough so that the grain boundaries act as an appreciable barrier to transport current flow, but the tilt angle of an individual grain is small enough to be neglected in computing the magnetization of the grain. For very weak magnetic fields where both the slab and the grains are never fully penetrated, the result of a rather long calculation shows that because of their interaction the two terms in the loss expression (4) are equal, and in ergs/cm^3

$$\frac{1}{V}\oint dt \int \mathbf{j}\cdot\mathbf{E}V = \frac{1}{V}\oint dt \int \mathbf{H}\cdot\dot{\mathbf{M}}dV = \frac{1}{24\pi}\frac{H^4{}_{a0}}{H_p^{grain}H_p^{slab}} \tag{7}$$

where the dot indicates a time derivative, H_{a0} is the peak value of a triangular applied field, while H_p^{slab} and H_p^{grain} are respectively the applied field required to fully penetrate the slab and the local field required to fully penetrate a grain. A fourth power dependence on H_{a0} has been measured by Ciszek et al[4] and also predicted by other authors[5], but based on particular mechanisms.

For a strong applied field, the internal magnetic fields of both $j_c^{\text{int}\,er}$ and $j_c^{\text{int}\,ra}$ can be neglected and therefore these currents do not interact. One obtains

(8)

$$\frac{1}{V}\oint dt \int \mathbf{j}\cdot\mathbf{E}dV \approx \frac{1}{2\pi}H_p^{slab}H_{a0}$$

$$\frac{1}{V}\oint dt \int \mathbf{H}\cdot\dot{\mathbf{M}}dV \approx \frac{1}{2\pi}H_p^{grain}H_{a0}. \tag{9}$$

similar to the result of Kwasnitza and Clerc[6]. The total loss depends on $H_p^{slab} + H_p^{grain}$, where the ratio will change with temperature and peak magnetic field because the value of j_c^{inter} used in Bean approximation must change. For intermediate field cases an H_{a0}^3 loss is expected.

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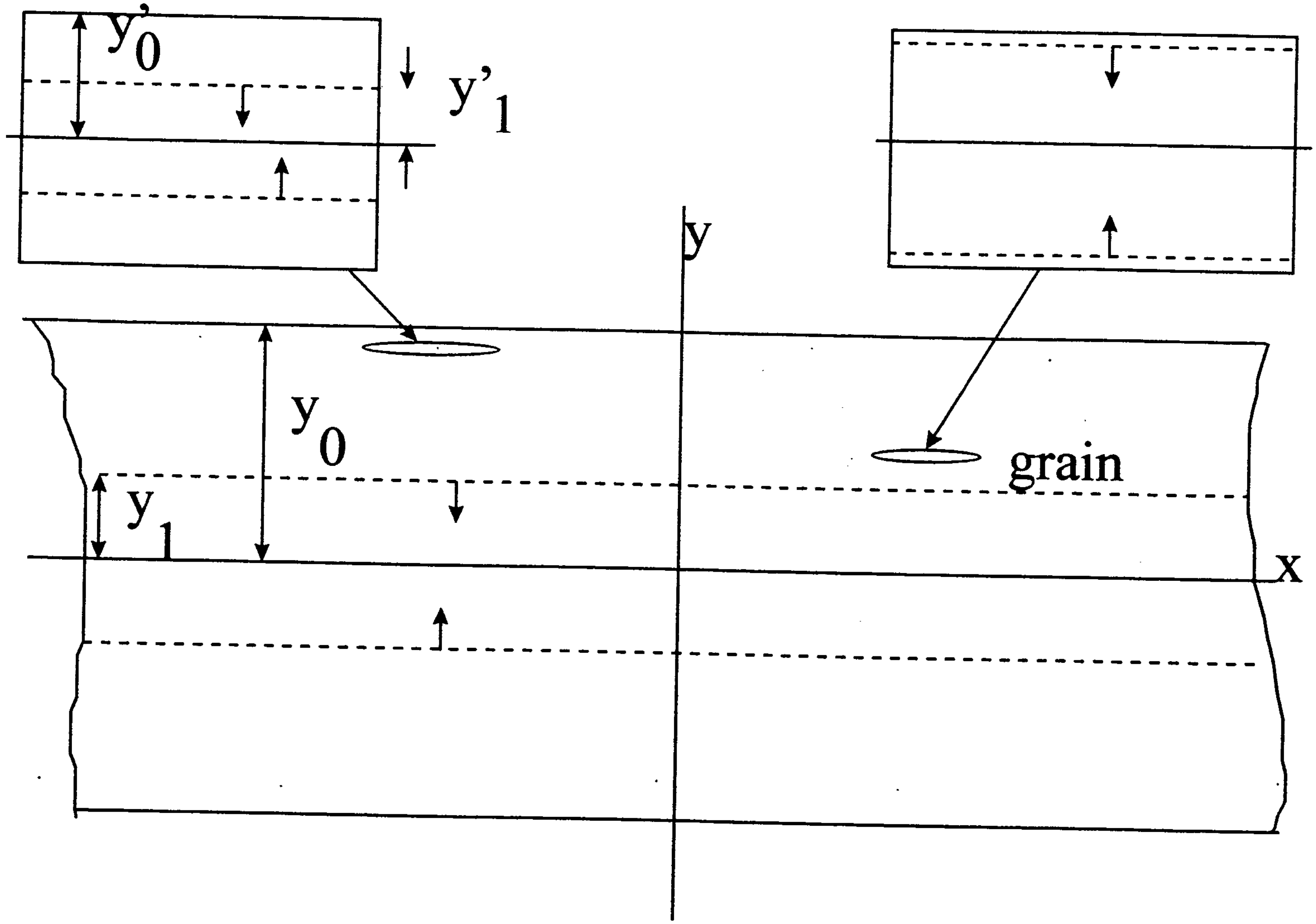
Figure caption

Fig. 1. A slab of half thickness y_0 showing two of the slab-like grains which fill the space. The magnetic field is applied in the z direction. Current flows along the x axis, and y_1 is the distance to the inwardly moving boundaries which reverse the current. The values of y with a prime have similar meaning for the grains. Note that grains near the surface are more fully penetrated than those near the moving boundary, and the magnetization loss requires an average over the grains.


y'0
y'1
y
y0
y1
grain
x

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Filamentary YBCO Conductors For AC Applications*

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High current density YBCO conductors as they are presently conceived for use at liquid nitrogen temperatures are in the form of tapes, usually the order of a centimeter in width and suitable only for dc use. Possibilities for developing similar YBCO conductors for ac applications are discussed, and the ac loss expected from such conductors is computed. The conversion from a dc to an ac conductor requires breaking up a wide-strip dc conductor into narrow strip-like filaments which spiral about the conductor axis. Various ways of producing this pattern are proposed.

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